CENTRE OF MASS, COLLISION

CENTRE OF MASS

- Centre of mass is a hypothetical point where all the masses are assumed to be concentrated under the given force.
- Position of centre of mass for n particles system is given

$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + ... + m_n\vec{r}_n}{m_1 + ... + m_n} = \frac{\text{Total linear moment of system}}{\text{Total mass}}$$

$$x_{CM} = \frac{m_1x_1 + ... + m_nx_n}{m_1 + ... + m_n} = \frac{\text{Total linear moment along x} - \text{direction}}{\text{Total mass}}$$

$$y_{CM} = \frac{m_1y_1 + ... + m_ny_n}{m_1 + ... + m_n} = \frac{\text{Total linear moment along y} - \text{axis}}{\text{Total mass}}$$

Position of Centre of mass for a rigid body is given by

$$\vec{r}_{CM} = \frac{\int \vec{r}dm}{\int dm} = \frac{\text{Total linear moment of the body}}{\text{Total mass}}$$

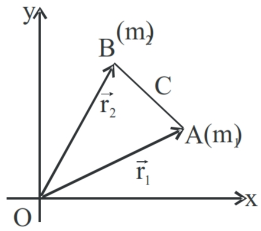
$$x_{CM} = \frac{\int xdm}{\int dm} = \frac{\text{Total linear moment along x} - \text{axis}}{\text{Total mass}}$$

$$y_{CM} = \frac{\int ydm}{\int dm} = \frac{\text{Total linear moment along y} - \text{axis}}{\text{Total mass}}$$

- Centre of mass (Mass Centre) is a point which always exist and it is unique. It might lie either inside or outside the body. If all the external forces are applied at the centre of mass, then there is only translational motion.
- Centre of mass of two particles system lie between the line joining of two masses.

$$\overrightarrow{OC} = \vec{r}_{CM} = \frac{m\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} = \vec{r}_2 - \frac{m\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$



and $\overrightarrow{CA} = -\dfrac{m_2\left(\vec{r}_2 - \vec{r}_1\right)}{m_1 + m_2}$

$$\frac{\overrightarrow{CA}}{\overrightarrow{CB}} = \frac{m_2}{m_1} \Rightarrow m_1\overrightarrow{CA} + m_2\overrightarrow{CB} = 0 \text{ and } \left|\frac{\overrightarrow{CA}}{\overrightarrow{CB}}\right| = \frac{m_2}{m_1}$$

and if $m_1 > m_2$, $\left|\overrightarrow{CA}\right| < \left|\overrightarrow{CB}\right|$, i.e. centre of mass lies closer to more mass.

Center of mass is the point about which total linear moment is zero

CENTRE OF MASS AND CENTRE OF GRAVITY

Centre of mass and centre of gravity are the same point if gravitational field is uniform, centre of gravity is a point in a body where the gravitational force acts on it, where body is balanced in all orientation.

Motion of Centre of mass : Consider n particles system of masses m_1, ..., m_n and position vectors $\vec{r}_1, ..., \vec{r}_n$ respectively.

$$M.\vec{r}_{CM} = m_1\vec{r}_1 + ... + m_n\vec{r}_n \quad ...(i) \qquad M = m_1 + ... + m_n = \text{Total mass of the system}$$

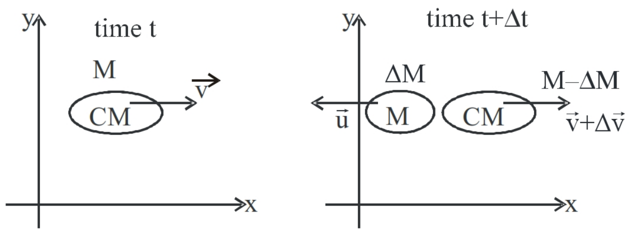
$$M.\vec{v}_{CM} = m_1\vec{v}_1 + ... + m_n\vec{v}_n \quad ...(ii)$$

$$M.\vec{a}_{CM} = m_1\vec{a}_1 + ... + m_n\vec{a}_n = \vec{F}_{Ext}^{(Total)} = M\frac{d\vec{v}_{CM}}{dt} \quad ...(iii)$$

For any system if there is no external force then $\vec{a}_{CM} = 0$ i.e. $\vec{v}_{CM}$ = constant.Hence for a system if system is at rest initially under no external force it means position of centre of mass remains the same during whole observation.

SYSTEM OF VARIABLE MASS SYSTEM

$$\vec{F}_{Ext} = \lim_{\Delta t \to 0}\frac{\overrightarrow{\Delta P}}{\Delta t} = \lim_{\Delta t \to 0}\frac{\vec{P}_f - \vec{P}_i}{\Delta t}$$



$$\vec{F}_{Ext} = \frac{\left[(M - \Delta M)\left(\vec{v} + \overrightarrow{\Delta v}\right) + \Delta M\vec{u}\right] - M\vec{v}}{\Delta t}$$

$$= M.\frac{\overrightarrow{\Delta v}}{\Delta t} + \left[\vec{u} - \left(\vec{v} + \overrightarrow{\Delta v}\right)\right].\frac{\Delta M}{\Delta t}$$

on $\Delta t \to 0$

$$\vec{F}_{Ext} = M.\frac{d\vec{v}}{dt} + (\vec{u} - \vec{v}).\left(-\frac{dM}{dt}\right);$$ (As mass is decreasing with time).

$$\Rightarrow M.\frac{d\vec{v}}{dt} = \vec{F}_{Ext} + \vec{u}_r\frac{dM}{dt}; \quad F_{thrust} = \vec{u}_r \,.\, \frac{d\mu}{dt}$$

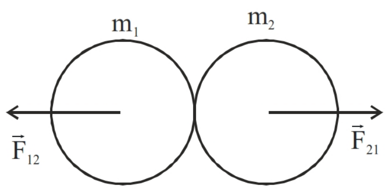
$$M.\frac{d\vec{v}}{dt} = \vec{F}_{Ext} + \vec{F}_{thrust}$$

$\vec{F}_{Ext}$ taken to be weight in case of rocket motion that can be neglecgted in comparision of thrust imparted to he rocket.

$\vec{F}_{thrust} = u_r\frac{dM}{dt}$ = This is momentum transfered by the ejected gas to the system per second.

COLLISION

A process in which there is a mutual foce of intraction of large magnitude for relatively small time.



$$\Delta\vec{P}_1 = \vec{F}_{12}\Delta t \text{ and } \Delta\vec{P}_2 = \vec{F}_{21}\Delta t$$

So, $\Delta\overrightarrow{P_1} + \Delta\overrightarrow{P_2} = \overrightarrow{F_{12}}\Delta t + \overrightarrow{F_{21}}\Delta t = 0 =$ Total impulse imparted to the system of colliding bodies and it implies total linear momentum remains conserved.If there is no external force acting on the system, the linear momentum of system is not changed by the collision.

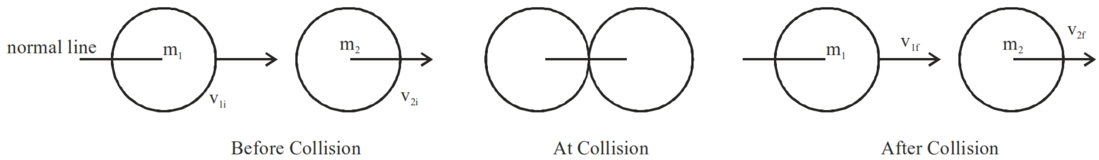
Elastic and inelastic collision

After collision colliding bodies come to their original shape and size and so no fraction of energy stored as potential energy hence kinetic energy remains conserved before and after collision.

Head on / oblique collision:

If direction of line of motion is along the line along which the impulse imparted, then collision is head-on otherwise oblique.

Head on elastic Collision



$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i} \quad ; \qquad v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}$$

Case I: If $m_1 = m_2$, then $v_{1f} = v_{2i}$ and $v_{2f} = v_{1i}$

After head-on elastic collision they simply exchange their velocities.

Case II: If $m_1 >> m_2$, $v_{2i} = 0$

then $v_{1f} = v_{1i}$ and $v_{2f} = 2v_{1i}$

Case III: If $m_1 << m_2$, $v_{2i} = 0$

then $v_{1f} = -v_{1i}$ and $v_{2f} = 0$

Newtons Experimental law of restitution

$$-e = \frac{\vec{v}_{2f} - \vec{v}_{1f}}{\vec{v}_{2i} - \vec{v}_{1i}} = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

After perfectly inelastic collision colliding bodies stick together or move together after collision.

$0 \le e \le 1$; e is the coefficient of restitution.

$e = 0$ for perfectly inelastic

$e = 1$ for perfectly elastic

For inelastic collision

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad \text{...(i)}$$

$$v_{2f} - v_{1f} = -e(v_{2i} - v_{1i}) \qquad \text{...(ii)}$$

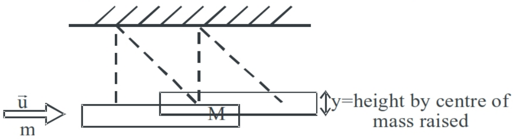
Solving, we have,

$$v_{1f} = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 + em_2}{m_1 + m_2}\right)v_{2i}; \qquad v_{2f} = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)v_{2i} + \left(\frac{m_1 + em_1}{m_1 + m_2}\right)v_{1i}$$

If $m_1 = m_2$ and $v_{2i} = 0$

then $v_{1f} = \left(\frac{1-e}{2}\right)v_{1i}$ and $v_{2f} = \left(\frac{1+e}{2}\right)v_{1i}$

Ballistic Pendulum (Perfectly inelastic collision)



u is the muzzle speed of bullet.

$$\frac{1}{2}mu^2 = \text{K.E. of bullet before collision}$$

$$(M+m)v = mu \Rightarrow \text{vel. of plank and bullet after collision} = \frac{mu}{M+m}$$

and $\quad \frac{1}{2}(M+m)v^2 = (M+m)g.y$ (Nelecting friction during the time bulled stops in plank)

or $\quad \frac{1}{2}(M+m)\left(\frac{mu}{M+m}\right)^2 = (M+m)g.y$

$$\therefore \frac{m^2u^2}{2(M+m)} = (M+m)gy\ ;\ u = \frac{M+m}{m}\sqrt{2gy}$$

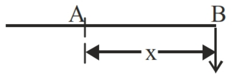
Fractional loss in K.E. in perfectly inelastic collision

$$\frac{\Delta K}{K} = \frac{\frac{1}{2}mu^2 - \frac{1}{2}(M+m)v^2}{\frac{1}{2}mu^2}$$

$$= \frac{\frac{1}{2}mu^2\left[1 - \frac{m}{M+m}\right]}{\frac{1}{2}mu^2} = \frac{M}{m+M}$$

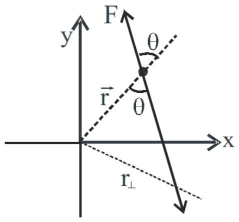
MOMENT OF THE FORCE/ TORQUE

Moment of the force about a point is the measure of rotation which the product of the applied force and the normal (i.e. perpendicular) distance of line of application of this force from this point. It is also called torque.



Moment of force F about point O is given by $\vec{r} \times \vec{p}$ which indicates the rotating tendency of the force about a point. The anticlockwise forces are generally taken as positive whereas clockwise forces are taken as negative. Torque is another name for moment of the force. Mathematically, torque $\vec{\tau} = \vec{r} \times \vec{F}$, $\theta |\vec{\tau}| = (r\sin\theta)F$

where F is applied force and $r\sin\theta$ is perpendicular to the line of force called moment area.



Torque is a vector. Its direction can be found out from right handed screw rule.

SI Unit of torque is Nm. Dimensional formula of torque is $[ML^2T^{-2}]$.

RELATIONS OF TORQUE IN DIFFERENT FORMS

In Cartesian co-ordinate system

$$\tau_x = yF_z - zF_y\ ;\ \tau_y = yF_x - xF_z \ \text{ and } \ \tau_z = xF_y - yF_x$$

In Polar co-ordinate system

$$\tau = (F\sin\theta)r,$$

where θ is the angle between the force F and the position vector r

In vector form

$$\vec{\tau} = \left(\hat{i}x + \hat{j}y + \hat{k}z\right) \times \left(\hat{i}F_x + \hat{j}F_y + \hat{k}F_z\right) = \vec{r} \times \vec{F}$$